# Chapter - Gravitation

## Topic-1: Kepler's Laws of Planetary Motion

### 1 MCQs with One Correct Answer

1. A binary star system consists of two stars $A$ and $B$ which have time period $T_A$ and $T_B$, radius $R_A$ and $R_B$ and mass $M_A$ and $M_B$. Then **[2006 - 3M, –1]**

   (a) if $T_A > T_B$ then $R_A > R_B$

   (b) if $T_A > T_B$ then $M_A > M_B$

   (c) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$

   (d) $T_A = T_B$

2. If the distance between the earth and the sun were half its present value, the number of days in a year would have been **[1996 - 2 Marks]**

   (a) 64.5 (b) 129

   (c) 182.5 (d) 730

### 6 MCQs with One or More than One Correct Answer

3. Imagine a light planet revolving around a very massive star in a circular orbit of radius $R$ with a period of revolution $T$. If the gravitational force of attraction between the planet and the star is proportional to $R^{-5/2}$ **[1989 - 2 Mark]**

   (a) $T^2$ is proportional to $R^3$

   (b) $T^2$ is proportional to $R^{7/2}$

   (c) $T^2$ is proportional to $R^{3/2}$

   (d) $T^2$ is proportional to $R^{3/73}$

### 10 Subjective Problems

4. Two satellites $S_1$ and $S_2$ revolve round a planet in coplanar circular orbits in the same sense. Their periods of revolution are 1 hour and 8 hours respectively. The radius of the orbit of $S_1$ is $10^4$ km. When $S_2$ is closest to $S_1$, find

   (i) the speed of $S_2$ relative to $S_1$,

   (ii) the angular speed of $S_2$ as actually observed by an astronaut in $S_1$. **[1986 - 6 Marks]**

## Topic-2: Acceleration due to Gravity

### 1 MCQs with One Correct Answer

1. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and a linear mass density $10^{-3}$ kg m$^{-1}$ into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth $= 6 \times 10^6$ m and the acceleration due to gravity on Earth is 10 ms$^{-2}$) **[Adv. 2014]**

   (a) 96 N (b) 108 N (c) 120 N (d) 150 N

### 4 Fill in the Blanks

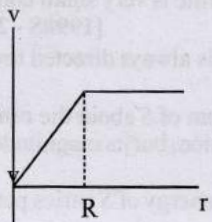
2. The numerical value of the angular velocity of rotation of the earth should be ............rad/s in order to make the effective acceleration due to gravity equal to zero. **[1984 - 2 Marks]**

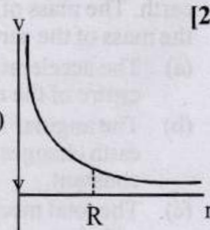
## Topic-3: Gravitational Field, Potential and Potential Energy

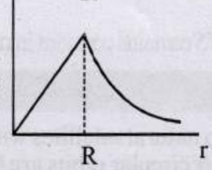
### 1 MCQs with One Correct Answer

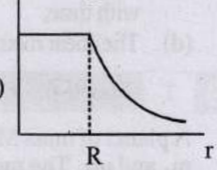
1. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$

where $\rho_0$ is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed v as a function of distance r $(0 < r < \infty)$ from the centre of the system is represented by – **[2008]**

(a) 


(b) 


(c) 


(d) 


### 6 MCQs with One or More than One Correct Answer

2. The magnitudes of the gravitational field at distance $r_1$ and $r_2$ from the centre of a uniform sphere of radius $R$ and mass $m$ are $F_1$ and $F_2$ respectively. Then: **[1994 - 2 Marks]**

(a) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 < R$ and $r_2 < R$

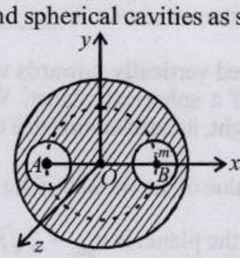
(b) $\frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$ if $r_1 > R$ and $r_2 > R$

(c) $\frac{F_1}{F_2} = \frac{r_1}{r_2}$ if $r_1 > R$ and $r_2 > R$

(d) $\frac{F_1}{F_2} = \frac{r_1^2}{r_2^2}$ if $r_1 < R$ and $r_2 < R$

3. A solid sphere of uniform density and radius 4 units is located with its centre at the origin $O$ of coordinates. Two spheres of equal radii 1 unit, with their centres at $A$ (–2, 0 ,0) and B (2, 0, 0) respectively, are taken out of the solid leaving behind spherical cavities as shown in fig

Then : **[1993-2 Marks]**




(a) The gravitational force due to this object at the origin is zero.

(b) the gravitational force at the point B (2, 0 ,0) is zero.

(c) the gravitational potential is the same at all points of circle $y^2 + z^2 = 36$.

(d) the gravitational potential is the same at all points on the circle $y^2 + z^2 = 4$.

## Topic-4: Motion of Satellites, Escape Speed and Orbital Velocity

### 1 MCQs with One Correct Answer

1. A particle of mass m is under the influence of the gravitational field of a body of mass M ($>> m$). The particle is moving in a circular orbit of radius $r_0$ with time period $T_0$ around the mass M. Then, the particle is subjected to an additional central force, corresponding to the potential energy $V_c(r) = \frac{m\alpha}{r^3}$, where α is a positive constant of suitable dimensions and $r$ is the distance from the center of the orbit. If the particle moves in the same circular orbit of radius $r_0$ in the combined gravitational potential due to M and $V_c(r)$, but with a new time period $T_1$, then $(T_1^2 - T_0^2)/T_1^2$ is given by

[G is the gravitational constant.] **[Adv. 2024]**

(a) $\frac{3\alpha}{GM r_0^2}$ (b) $\frac{\alpha}{2GM r_0^2}$

(c) $\frac{\alpha}{GM r_0^2}$ (d) $\frac{2\alpha}{GM r_0^2}$

2. Two satellites P and Q are moving in different circular orbits around the Earth (radius R). The heights of P and Q from the Earth surface are $h_P$ and $h_Q$, respectively, where $h_P = R/3$. The accelerations of P and Q due to Earth's gravity are $g_P$ and $g_Q$, respectively. If $g_P/g_Q = 36/25$, what is the value of $h_Q$? **[Adv. 2023]**

(a) 3R/5 (b) R/6 (c) 6R/5 (d) 5R/6

3. What is the minimum energy required to launch a satellite of mass m from the surface of a planet of mass M and radius R in a circular orbit at an altitude of 2R? **[2012]**

(a) $\frac{5GmM}{6R}$ (b) $\frac{2GmM}{3R}$

(c) $\frac{GmM}{2R}$ (d) $\frac{GmM}{2R}$

4. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the Sun and the Earth. The Sun is $3 \times 10^5$ times heavier than the Earth and is at a distance $2.5 \times 10^4$ times larger than the radius of the Earth. The escape velocity from Earth's gravitational field

is $v_e = 11.2\ km\ s^{-1}$. The minimum initial velocity ($v_s$) required for the rocket to be able to leave the Sun–Earth system is closest to (Ignore the rotation and revolution of the Earth and the presence of any other planet) **[Adv. 2017]**

(a) $v_s = 22\ km\ s^{-1}$ (b) $v_s = 42\ km\ s^{-1}$
(c) $v_s = 62\ km\ s^{-1}$ (d) $v_s = 72\ km\ s^{-1}$

## 2 Integer Value Answer

**5.** Two spherical stars $A$ and $B$ have densities $\rho_A$ and $\rho_B$, respectively. $A$ and $B$ have the same radius, and their masses $M_A$ and $M_B$ are related by $M_B = 2M_A$. Due to an interaction process, star $A$ loses some of its mass, so that its radius is halved, while its spherical shape is retained, and its density remains $\rho_A$. The entire mass of lost by $A$ is deposited as a thick spherical shell on $B$ with the density of the shell being $\rho_A$. If $v_A$ and $v_B$ are the escape velocities from $A$ and $B$ after the interaction process, the ratio $\frac{v_B}{v_A} = \sqrt{\frac{10n}{15^{1/3}}}$ the value of $n$ is ______. **[Adv. 2022]**

**6.** A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is $\frac{1}{4}$th of its value of the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[Adv. 2015]**

**7.** Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$. where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the earth. If the escape speed on the surface of the earth is taken to be $11\ kms^{-1}$, the escape speed on the surface of the planet in $kms^{-1}$ will be **[2010]**

## 4 Fill in the Blanks

**8.** A particle is projected vertically upwards from the surface of earth (radius $R_e$) with a kinetic energy equal to half of the minimum value needed for it to escape. The height to which it rises above the surface of earth is.... **[1997 - 2 Marks]**

**9.** The masses and radii of the Earth and the Moon are $M_1$, $R_1$ and $M_2$, $R_2$ respectively. Their centres are at a distance $d$ apart. The minimum speed with which a particle of mass $m$ should be projected from a point midway between the two centres so as to escape to infinity is ................ **[1988 - 2 Marks]**

**10.** A geostationary satellite is orbiting the earth at a height of $6R$ above the surface of the earth, where $R$ is the radius of the earth. The time period of another satellite at a height of $2.5R$ from the surface of the earth is .........hours. **[1987 - 2 Marks]**

## 5 True / False

**11.** It is possible to put an artificial satellite into orbit in such a way that it will always remain directly over New Delhi. **[1984 - 2 Marks]**

## 6 MCQs with One or More than One Correct Answer

**12.** Two spherical planets $P$ and $Q$ have the same uniform density $\rho$, masses $M_P$ and $M_Q$ and surface areas $A$ and $4A$ respectively. A spherical planet $R$ also has uniform density $\rho$ and its mass is $(M_P + M_Q)$. The escape velocities from the planets $P$, $Q$ and $R$ are $V_P$, $V_Q$ and $V_R$, respectively. Then **[2012]**

(a) $V_Q > V_R > V_P$ (b) $V_R > V_Q > V_P$
(c) $V_R / V_P = 3$ (d) $V_P / V_Q = \frac{1}{2}$

**13.** A satellite $S$ is moving in an elliptical orbit around the earth. The mass of the satellite is very small compared to the mass of the earth. **[1998S - 2 Marks]**

(a) The acceleration of $S$ is always directed towards the centre of the earth.
(b) The angular momentum of $S$ about the centre of the earth changes in direction, but its magnitude remains constant.
(c) The total mechanical energy of $S$ varies periodically with time.
(d) The linear momentum of $S$ remains constant in magnitude.

## 7 Match the Following

**14.** A planet of mass M, has two natural satellites with masses $m_1$ and $m_2$. The radii of their circular orbits are $R_1$ and $R_2$ respectively, Ignore the gravitational force between the satellites. Define $v_1$, $L_1$, $K_1$ and $T_1$ to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1; and $v_2$, $L_2$, $K_2$, and $T_2$ to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[Adv. 2018]**

| LIST–I | LIST–II |
|---|---|
| **P.** $v_1/v_2$ | **1.** 1/8 |
| **Q.** $L_1/L_2$ | **2.** 1 |
| **R.** $K_1/K_2$ | **3.** 2 |
| **S.** $T_1/T_2$ | **4.** 8 |

(a) $P \to 4; Q \to 2; R \to 1; S \to 3$
(b) $P \to 3; Q \to 2; R \to 4; S \to 1$
(c) $P \to 2; Q \to 3; R \to 1; S \to 4$
(d) $P \to 2; Q \to 3; R \to 4; S \to 1$

## 9 Assertion and Reason Type Questions

**15.** **STATEMENT - 1 :** An astronaut in an orbiting space station above the earth experiences weightlessness.

**because**

**STATEMENT - 2 :** An object moving around the earth under the influence of Earth's gravitational force is in a state of "free-fall". **[2008]**

(a) Statement- 1 is True, Statement-2 is True, Statement-2 is a correct explanation for Statement -1
(b) Statement -1 is True, Statement -2 is True ; Statement-2 is NOT a correct explanation for Statement - 1
(c) Statement - 1 is True, Statement- 2 is False
(d) Statement -1 is False, Statement -2 is True

### 10 Subjective Problems

**16.** A body is projected vertically upwards from the bottom of a crater of moon of depth $\frac{R}{100}$ where $R$ is the radius of moon with a velocity equal to the escape velocity on the surface of moon. Calculate maximum height attained by the body from the surface of the moon. **[2003 - 4 Marks]**

**17.** Distance between the centres of two stars is $10a$. The masses of these stars are $M$ and $16M$ and their radii $a$ and $2a$, respectively. A body of mass $m$ is fired straight from the surface of the larger star towards the smaller star. What should be its minimum initial speed to reach the surface of the smaller star? Obtain the expression in terms of $G$, $M$ and $a$. **[1996 - 5 Marks]**

**18.** An artificial satellite is moving in a circular orbit around the earth with a speed equal to half the magnitude of escape velocity from the earth. **[1990 - 8 Mark]**

(i) Determine the height of the satellite above the earth's surface.

(ii) If the satellite is stopped suddenly in its orbit and allowed to fall freely onto the earth, find the speed with which it hits the surface of the earth.

**19.** Three particles, each of mass $m$, are situated at the vertices of an equilateral triangle of side length a. The only forces acting on the particles are their mutual gravitational forces. It is desired that each particle moves in a circle while maintaining the original mutual separation $a$. Find the intial velocity that should be given to each particle and also the time period of the circular motion. **[1988 - 5 Marks]**

## Topic-5: Miscellaneous (Mixed Concepts) Problems

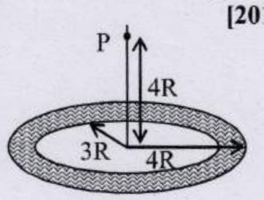
### 1 MCQs with One Correct Answer

**1.** Consider a spherical gaseous cloud of mass density $\rho(r)$ in a free space where r is the radical distance from its center. The gaseous cloud is made of particles of equal mass m moving in circular orbits about the common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If $\rho(r)$ is constant in time. The particle number density $n(r) = \rho(r)/m$ is
[G is universal gravitational constant] **[Adv. 2019]**

(a) $\frac{3K}{\pi r^2 m^2 G}$ (b) $\frac{K}{2\pi r^2 m^2 G}$

(c) $\frac{K}{\pi r^2 m^2 G}$ (d) $\frac{K}{6\pi r^2 m^2 G}$

**2.** A thin uniform annular disc (see figure) of mass $M$ has outer radius $4R$ and inner radius $3R$. The work required to take a unit mass from point P on its axis to infinity is **[2010]**




(a) $\frac{2GM}{7R}(4\sqrt{2}-5)$

(b) $-\frac{2GM}{7R}(4\sqrt{2}-5)$

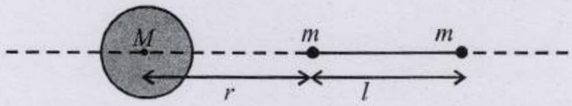
(c) $\frac{GM}{4R}$ (d) $\frac{2GM}{5R}(\sqrt{2}-1)$

### 2 Integer/Non-negative Integer Value Answer

**3.** The distance between two stars of masses $3M_s$ and $6M_s$ is $9R$. Here $R$ is the mean distance between the centers of the Earth and the Sun, and $M_s$ is the mass of the Sun. The two stars orbit around their common center of mass in circular orbits with period $nT$, where $T$ is the period of Earth's revolution around the Sun. The value of $n$ is ____.

**[Adv. 2021]**

**4.** A large spherical mass $M$ is fixed at one position and two identical point masses $m$ are kept on a line passing through the centre of $M$ (see figure). The point masses are connected by a rigid massless rod of length $\ell$ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to $M$ is at a distance $r = 3\ell$ from $M$, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of $k$ is **[Adv. 2015]**




### 6 MCQs with One or More than One Correct Answer

**5.** Two bodies, each of mass M, are kept fixed with a separation 2L. A particle of mass m is projected from the midpoint of the line joining their centres, perpendicular to the line. The gravitational constant is G. The correct statement(s) is (are) **[Adv. 2013]**

(a) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $4\sqrt{\frac{GM}{L}}$

(b) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $2\sqrt{\frac{GM}{L}}$

(c) The minimum initial velocity of the mass m to escape the gravitational field of the two bodies is $\sqrt{\frac{2GM}{L}}$

(d) The energy of the mass m remains constant

## Answer Key

**Topic-1 : Kepler's Laws of Planetary Motion**

1. (d) 2. (b) 3. (b)

**Topic-2 : Acceleration due to Gravity**

1. (b) 2. ($1.24 \times 10^{-3}$ rad/s)

**Topic-3 : Gravitational Field, Potential and Potential Energy**

1. (c) 2. (a,b) 3. (a,c,d)

**Topic-4 : Motion of Satellites, Escape Speed and Orbital Velocity**

1. (a) 2. (a) 3. (a) 4. (b) 5. (2.3) 6. (2) 7. (3) 8. (R)

9. $\left(\sqrt{\frac{4G}{d}(M_1+M_2)}\right)$ 10. (8.48 h)

10. False 11. (b, d) 12. (a) 13. (b) 14. (a)

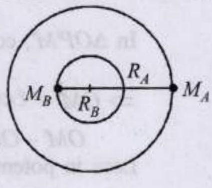
**Topic-5 : Miscellaneous (Mixed Concepts) Problems**

1. (b) 2. (a) 3. (9) 4. (7) 5. (b, d)

# Hints & Solutions

## Topic-1: Kepler's Laws of Planetary Motion

**1.** **(d)** Incase of binary star system, the gravitational force of attraction between the stars will provide the necessary centripetal forces.




So angular velocity $\omega$ of both stars is the same. Therefore time period $T$

$= \frac{2\pi}{\omega}$ remains the same.

**2.** **(b)** According to Kepler's law $T^2 \propto R^3$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}$$

Here $T_1 = 365$ days ; $T_2 = ?$; $R_1 = R$ and $R_2 = \frac{R}{2}$

$$\Rightarrow \quad T_2 = T_1\left(\frac{R_2}{R_1}\right)^{3/2} = 365\left[\frac{R/2}{R}\right]^{3/2} = 129 \text{ days}$$

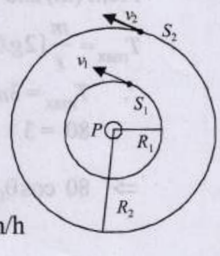
**3.** **(b)** The centripetal force is provided by the gravitational force of attraction

So, $mR\omega^2 = GMmR^{-5/2}$

$$\Rightarrow \frac{mR \times 4\pi^2}{T^2} = \frac{GMm}{R^{5/2}} \Rightarrow T^2 \propto R^{7/2}$$

**4.** (i) According to Kepler's third law of planetary motion. $T^2 \propto R^3$

$$\therefore \quad \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow R_2^3 = R_1^3 \times \frac{T_2^2}{T_1^2}$$




$$\Rightarrow \quad R_2^3 = \left(10^4\right)^3 \times \frac{8^2}{1^2} = 64 \times 10^{12}$$

$\Rightarrow \quad R_2 = 4 \times 10^4$ km.

Linear speed of satellite $S_1$

$$v_1 = \frac{2\pi R_1}{T_1} = \frac{2\pi \times 10^4}{1} = 2\pi \times 10^4 \text{ km/h}$$

Linear speed of satellite $S_2$,

$$v_2 = \frac{2\pi R_2}{T_2} = \frac{(2\pi)(4 \times 10^4)}{8} = \pi \times 10^4 \text{ km/h}$$

Hence the speed of satellite $S_2$ w.r.t. $S_1$

$= v_2 - v_1 = \pi \times 10^4 - 2\pi \times 10^4 = -\pi \times 10^4$ km/h

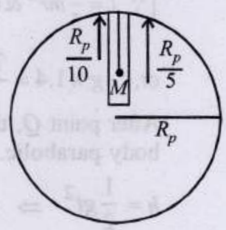
(ii) Angular speed of $S_2$ w.r.t. $S_1$

$$= \frac{v_2 - v_1}{R_2 - R_1} = \frac{3.14 \times 10^4 \times 5/18}{3 \times 10^4 \times 10^3} = 3 \times 10^{-4} \text{ rad/s}$$

## Topic-2: Acceleration due to Gravity

**1.** **(b)** The mass of the wire

$= 10^{-3} \times 1.2 \times 10^5 = 120$ kg




Acceleration due to gravity at the surface of the planet and earth

$$g_p = \frac{4}{3}\pi\rho G R_P \,;$$

and $g_e = \frac{4}{3}\pi\rho G R_E$

$$\therefore \quad \frac{g_p}{g_e} = \frac{R_p}{R_E} = \frac{1}{10} \Rightarrow g_p = \frac{10}{10} = 1 \text{ ms}^{-2}$$

Let $g_{pM}$ be the acceleration due to gravity at point M which is the mid point of the wire and is at a depth of $\frac{R_p}{10}$.

$$g_{pM} = g_p\left[1 - \frac{R_p/10}{R_p}\right] = 1[1 - 0.1] = 0.9 \text{ ms}^{-2}$$

$\therefore$ Force = mass of wire $\times g_{pM} = 120 \times 0.9 = 108$ N

**2.** **($1.24 \times 10^{-3}$ rad/s)**

Using $\quad g' = g - R\omega^2 \cos^2\phi$

At equator, $\quad \phi = 0, \ \therefore g' = g - R\omega^2$

Here $g' = 0 \quad \therefore \omega = \sqrt{\frac{g}{R}} = 1.24 \times 10^{-3}$ rad/s

## Topic-3: Gravitational Field, Potential and Potential Energy

**1.** **(c)** For $r \geq R$

$\frac{mv^2}{r} = m \times \left[\frac{GM}{r^2}\right]$ where $M$ is the total mass of the spherical system.

$$\therefore \quad v \propto \frac{1}{\sqrt{r}} \qquad \left(\because M = \left(\frac{4}{3}\pi R^3\right)\rho_0\right)$$

**For r < R**

$$\therefore \quad \frac{mv^2}{r} = m\left[\frac{GMr}{R^3}\right] \Rightarrow v \propto r$$

i.e., $v - r$ graph is a straight line passing through origin.

**2.** **(a,b)** For $r > R$, the gravitational field, $F = \frac{GMm}{r^2}$

$$\therefore \quad F_1 = \frac{GMm}{r_1^2} \text{ and } F_2 = \frac{GMm}{r_2^2} \text{ or, } \frac{F_1}{F_2} = \frac{r_2^2}{r_1^2}$$

For $r < R$, the gravitational field, $F = \frac{Gm}{R^3} \times r$

$\therefore\ F_1 = \frac{GMm}{R^3}\times r_1$ and $F_2 = \frac{GMm}{R^3}\times r_2$

$\therefore\ \frac{F_1}{F_2} = \frac{r_1}{r_2}$

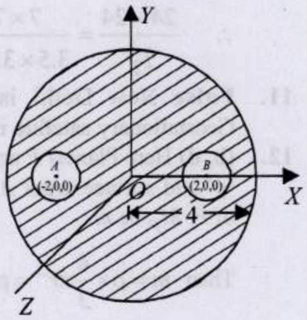
**3.** **(a,c,d)** The gravitational field (E) intensity at the point $O$ i.e., centre of a solid sphere is zero. Force acting on a test mass $m_0$ placed at $O$

$$F = m_0 E = m_0 \times 0 = 0$$

The gravitational field due to masses at A and B at 'O' is equal and opposite.

Now, $y^2 + z^2 = 36$ represents the equation of a circle with centre (0, 0, 0) and radius 6 units the plane of the circle is perpendicular to $x$-axis.

As the plane of these circles is Y-Z ⊥ to X-axis so potential at any point on these two circles will be constant due to mass $M$ and masses at $A$ and $B$.




## Topic-4: Motion of Satellites, Escape Speed and Orbital Velocity

**1.** **(a)** Particle of mass m is moving in a circular orbit of radius $r_0$ with time period $T_0$ around the mass M provides centripetal force

$$F_c = \frac{mv^2}{r_0}$$

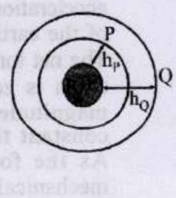
$$\frac{GMm}{r_0^2} - \frac{3m\alpha}{r_0^4} = \frac{mv^2}{r_0}$$

or, $\frac{\omega_1^2}{\omega_0^2} = \frac{\frac{GM}{r_0^2} - \frac{3\alpha}{r_0^4}}{\frac{GM}{r_0^2}}$

or, $\frac{T_0^2}{T_1^2} = 1 - \frac{3\alpha}{GMr_0^2} \Rightarrow \frac{T_1^2 - T_0^2}{T_1^2} = \frac{3\alpha}{GMr_0^2}$

**2.** **(a)** Acceleration due to gravity 'g' varies with height

$$h = \frac{GM}{(R+h)^2}$$




$$\frac{g_P}{g_Q} = \frac{\frac{GM}{(R+h_P)^2}}{\frac{GM}{(R+h_Q)^2}} \quad \therefore \frac{36}{25} = \frac{(R+h_Q)^2}{(R+h_P)^2}$$

$$\therefore R + h_Q = \frac{6}{5}\left(R + \frac{R}{3}\right) \quad \left[\because h_P = \frac{R}{3}\text{ given}\right]$$

$$\therefore h_Q = \frac{24}{15}R - R = \frac{9}{15}R = \frac{3}{5}R$$

**3.** **(a)** As we know,

Gravitational potential energy $= \frac{-GMm}{r}$

and orbital velocity, $v_0 = \sqrt{GM/R+h}$

$$E_f = \frac{1}{2}mv_0^2 - \frac{GMm}{3R} = \frac{1}{2}m\frac{GM}{3R} - \frac{GMm}{3R}$$

$$= \frac{GMm}{3R}\left(\frac{1}{2} - 1\right) = \frac{-GMm}{6R} \Rightarrow E_i = \frac{-GMm}{R} + K$$

$E_i = E_f$

Therefore minimum required energy, $K = \frac{5GMm}{6R}$

**4.** **(b)** Applying energy conservation

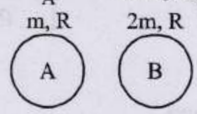
$$\frac{1}{2}mV_s^2 - \frac{GM_e m}{R_e} = \frac{GM_e m \times 3\times 10^5}{2.5\times 10^4 R_e}$$

$$\frac{V_s^2}{2} = \frac{GM_e}{R_e}\left[1 + \frac{3\times 10^5}{2.5\times 10^4}\right]$$

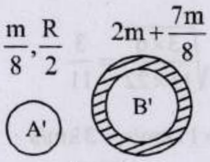
$$V_s = \sqrt{13\left(\frac{2GM_e}{R_e}\right)} \left(\because V_e = \sqrt{\frac{2Gm_e}{R_e}} = 11.2\text{ km/s}\right)$$

$$\therefore\ V_s = \sqrt{13}\times 11.2 \approx 42\text{ m/s}$$

**5.** **(2.3)** Initially, let $M_A = m$. Then, $M_B = 2m$




Finally,




Now, $V_e = \sqrt{\frac{2GM}{R}}$

$$\frac{V_B}{V_A} = \sqrt{\frac{M'_B}{M'_A}\times\frac{R'_A}{R'_B}} = \sqrt{\frac{2m + \frac{7m}{8}}{\frac{m}{8}}\times\frac{\frac{R}{2}}{R'_B}}$$

Now, $\rho\times\frac{4}{3}\pi\left((R'_B)^3 - R^3\right) = \frac{7}{8}\times\rho\times\frac{4}{3}\pi R^3$

$$\Rightarrow (R'_B)^3 - R^3 = \frac{7}{8}R^3 \Rightarrow (R'_B)^3 = \frac{15}{8}R^3$$

$$\Rightarrow R'_B = \frac{15^{1/3}}{2}R$$

So, $\frac{V_B}{V_A} = \sqrt{\frac{\frac{23m}{8}}{\frac{m}{8}}\times\frac{\frac{R}{2}}{\frac{15^{1/3}R}{2}}} = \sqrt{\frac{\frac{23m}{8}}{\frac{m}{8}}\times\frac{1}{15^{1/3}}}$

$$= \sqrt{\frac{23}{15^{1/3}}} = \sqrt{\frac{10\times 2.3}{15^{1/3}}}$$

Therefore, n = 2.3

6. **(2)** Let $h$ be the height to which the bullet rises with the height acceleration due to gravity varies as

$$g^1 = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow \frac{g}{4} = g\left(1+\frac{h}{R}\right)^{-2} \Rightarrow h = R$$

We know escape speed, $v_e = \sqrt{\frac{2GM}{R}} = v\sqrt{N}$ (given) ...(i)

Now applying conservation of energy principle

Loss of kinetic energy = gain in gravitaional potential energy

$$\therefore \frac{1}{2}mv^2 = -\frac{GMm}{2R} - \left(-\frac{GMm}{R}\right)$$

$$\therefore v = \sqrt{\frac{GM}{R}} \quad ...(ii)$$

Comparing eq. (i) & (ii) we get $N = 2$

7. **(3)** We know that escape speed $v = \sqrt{2gR}$

$$\therefore \frac{v_p}{v_e} = \sqrt{\frac{g_p}{g} \times \frac{R_p}{R}} \quad ...(i)$$

Given $\frac{g_p}{g_e} = \frac{\sqrt{6}}{11}$ ...(ii)

Also $g = \frac{4}{3}\pi G\rho R \quad \therefore \frac{g_p}{g_e} = \frac{\rho_p}{\rho} \times \frac{R_p}{R}$

$$\therefore \frac{\sqrt{6}}{11} = \frac{2}{3} \times \frac{R_p}{R} \quad \left[\because \frac{\rho_p}{\rho} = \frac{2}{3}\text{(given)}\right]$$

$$\therefore \frac{R_p}{R} = \frac{3\sqrt{6}}{22} \quad ...(iii)$$

From (i), (ii) & (iii)

$$\frac{v_p}{v_e} = \sqrt{\frac{\sqrt{6}}{11} \times \frac{3\sqrt{6}}{22}} = \sqrt{\frac{3\times 6}{11\times 22}} = \frac{3}{11}$$

$$\therefore v_p = \frac{3}{11} \times v_e = \frac{3}{11} \times 11\,\text{km/s} = 3\,\text{km/s}$$

8. **(R)** From conservation of mechanical energy

$$\frac{1}{2}mv^2 + \left(-\frac{GMm}{R}\right) = -\frac{GMm}{(R+h)}$$

KE provided $= \frac{1}{2} \times$ KE of escape

or $\frac{1}{2}mv^2 = \frac{1}{2} \times \frac{GMm}{R} \quad \left[\because V_e = \sqrt{\frac{GM}{R} \times 2}\right]$

$$\therefore \frac{GMm}{2R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$$\Rightarrow -\frac{1}{2R} = -\frac{1}{R+h} \Rightarrow R + h = 2R \text{ or, } h = R$$

9. $\left(\sqrt{\frac{4G}{d}(M_1 + M_2)}\right)$ Total mechanical energy mass $m$ at a midway point between the centres of earth and moon

$$= \frac{GM_1m}{d/2} - \frac{GM_2m}{d/2} + \frac{1}{2}mV_e^2$$

$$= \frac{Gm}{d/2}(M_1 + M_2) + \frac{1}{2}mV_e^2 \therefore \frac{1}{2}mV_e^2 = \frac{Gm}{d/2}(M_1 + M_2)$$

[$\because$ final mechanical energy is zero]

where $V_e$ is the velocity with which mass $m$ is projected.

$$\Rightarrow V_e = \sqrt{\frac{4G}{d}(M_1 + M_2)}$$

10. **(8.48 h)** According to Kepler's law $T^2 \propto R^3$

$$\therefore \frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3}.$$

Here $R_1 = R + 6R = 7R$ and $R_2 = 2.5R + R = 3.5\,R$

$T_1 = 24$ hours

$$\therefore \frac{24 \times 24}{T_2^2} = \frac{7\times 7\times 7\times R^3}{3.5\times 3.5\times 3.5\times R^3} \Rightarrow T_2 = 8.48\,\text{h}$$

11. **False** New Delhi is not on the equatorial plane. Geostationary satellite is launched on the equatorial plane.

12. **(b, d)** Here Planets $P$ and $Q$ have the same uniform density '$\rho$' and surface areas $A$ and $4A$ respectively. Let the mass of $P$, $M_P$ be $m$.

Then $m = \rho \times \frac{4}{3}\pi r^3 = \rho \times \frac{4}{3}\pi\left[\frac{A}{4\pi}\right]^{3/2}$

The mass of $M_Q = \rho \times \frac{4}{3}\pi\left[\frac{4A}{4\pi}\right]^{3/2} = 8\,\text{m}$

$\therefore$ The mass of Planet $R = 8\,\text{m} + \text{m} = 9\,\text{m}$

If the radius of $P = r$

Then the radius of $Q = 2r$

$$\left[\because r_Q = \left(\frac{4A}{4\pi}\right)^{3/2} = 2\left(\frac{A}{4\pi}\right)^{3/2}\right]$$

and radius of $R = 9^{1/3}r$ $\quad \left[\because M_R = M_P + M_Q,\ r_R^3 = r^3 + (2r)^3 = 9r^3\right]$

As we know, escape velocity from the planet

$$V_e = \sqrt{\frac{2GM}{R}} \quad \therefore v_P = \sqrt{\frac{2GM_P}{R_p}} = \sqrt{\frac{2Gm}{r}}$$

$$v_Q = \sqrt{\frac{2GM_Q}{R_Q}} = \sqrt{\frac{2G\,(8m)}{2r}} = 2v_P$$

$$v_R = \sqrt{\frac{2G\,(9\,m)}{9^{1/3}r}} = 9^{1/3}v_P$$

13. **(a)** Force on satellite is always towards earth which attracts the satellite with the gravitational force $F$, therefore, acceleration of satellite $S$ is always directed towards centre of the earth.

The net torque of this gravitational force $F$ about centre of earth is zero. Therefore, angular momentum (both in magnitude and direction) of $S$ about centre of earth is constant throughout.

As the force $F$ is conservative in nature, therefore mechanical energy of satellite remains constant. Speed of $S$ is maximum when it is nearest to earth and minimum when it is farthest.

14. **(b)** $\because$ Orbital velocity,

$$V = \sqrt{\frac{GM}{R}}, \text{ or, } V \propto \frac{1}{\sqrt{R}} \quad \therefore \frac{V_1}{V_2} = \sqrt{\frac{R_2}{R_1}} = \frac{2}{1}$$

$$\frac{L_1}{L_2}=\frac{m_1v_1R_1}{m_1v_2R_2}=\frac{2\times2\times1}{1\times1\times4}=\frac{1}{1}$$

Kinetic energy, $K=\frac{GMm}{2R}$.

$$\therefore\ \frac{k_1}{k_2}=\frac{m_1}{m_2}\times\frac{R_1}{R_2}=\frac{2\times4}{1\times1}=\frac{8}{1}$$

From Kepler's law of planetary motion.

$$T^2\propto R^3\ \therefore\ \frac{T_1}{T_2}=\left(\frac{R_1}{R_2}\right)^{3/2}=\frac{1}{8}$$

**15.** **(a)** An astronant in an orbiting space station above the earth experiences weight less as he is in a state of free fall. The force acting on astronant is utilised in providing necessary centripetal force.

**16.** Applying mechanical energy conservation,

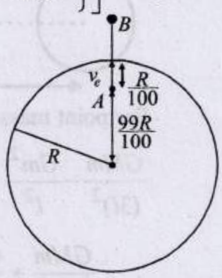
Total energy at $A$ = Total energy at $B$

$(K.E.)_A+(P.E.)_A=(P.E.)_B$

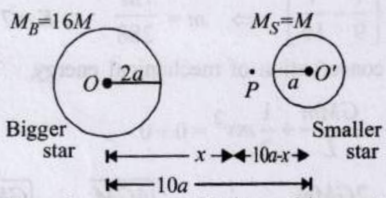
$$\Rightarrow\ \frac{1}{2}m\times\frac{2GM}{R}+\left[\frac{-GMm}{2R^3}\left\{3R^2-\left(\frac{99R}{100}\right)^2\right\}\right]=-\frac{GMm}{R+h}$$

Solving above equation, we get $h=99.5\,R$.

(Maximum height attained by the body from the surface of the earth)




**17.** Let the force of attraction is zero at a distance $x$ from the bigger star.




Then force on mass $m$ due to bigger star = force on mass $m$ due to small star

$$\frac{GM_Bm}{x^2}=\frac{GM_Sm}{(10a-x)^2}\Rightarrow\frac{16M}{x^2}=\frac{M}{(10a-x)^2}\Rightarrow x=8a$$

Let $v$ denote velocity with which the body of mass $m$ is fired so that it crosses $P$ along $OP$. The energy is conserved.

The energy of the system (of these masses) initially = final energy when $m$ is at $P$

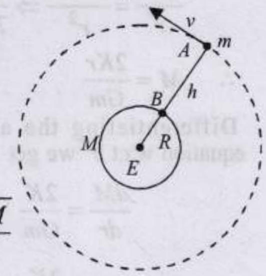
$$-\frac{GM_BM_S}{10a}-\frac{GM_Bm}{2a}-\frac{GM_Sm}{8a}+\frac{1}{2}mv^2$$

$$=-\frac{GM_BM_S}{10a}-\frac{GM_Bm}{8a}-\frac{GM_Sm}{2a}$$

$[\because M_B=16M;\ M_S=M]$

$$\therefore\ v=\frac{3}{2}\sqrt{\frac{5GM}{a}}$$

**18.** (i) Here centripetal force is provided by the gravitational pull




$$\therefore\ \frac{mv^2}{(R+h)}=\frac{GMm}{(R+h)^2}$$

$$\Rightarrow\ v^2=\frac{GM}{R+h}$$

Given $v=\frac{1}{2}v_e=\frac{1}{2}\sqrt{\frac{2GM}{R}}$

$$\therefore\ \frac{1}{4}\left(\frac{2GM}{R}\right)=\frac{GM}{R+h}\Rightarrow 2R+2h=4R\ \therefore\ h=R=6400\text{ km}.$$

(ii) Let V be the speed with which the satellite hits the surface of the earth. If the satellite is stopped, its kinetic energy is zero. When it falls freely on the Earth, its potential energy decreases and converts into kinetic energy.

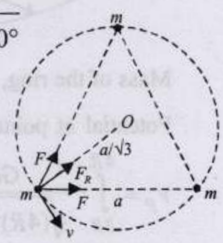
$\therefore\ (P.E.)_A-(P.E.)_B=K.E.$

$$\Rightarrow\ \frac{-GMm}{2R}-\left(\frac{-GMm}{R}\right)=\frac{1}{2}mv^2$$

$$\Rightarrow\ v=\sqrt{\frac{GM}{R}}=\sqrt{gR}=\sqrt{9.8\times6.4\times10^6}$$

$=7920\text{ m/s}=7.92\text{ km/s}$

**19.** Let the initial velocity that should be given to each particle be 'V'. The centripetal force is provided by the resultant gravitational attraction of the two masses.




$$F_R=\sqrt{F^2+F^2+2F^2\cos60^\circ}$$

$$=\sqrt{3}F=\sqrt{3}G\frac{m\times m}{a^2}$$

$$\therefore\ \sqrt{3}G\frac{m^2}{a^2}=\frac{mv^2}{r}$$

$\left(\frac{mv^2}{r}=\text{centripetal force}\right)$

Radius of the circular path $r=\frac{2}{3}\sqrt{a^2-\frac{a^2}{4}}=\frac{a}{\sqrt{3}}$

$$v^2=\frac{\sqrt{3}Gmr}{a^2}=\frac{\sqrt{3}Gma}{a^2\times\sqrt{3}}\Rightarrow v=\sqrt{\frac{Gm}{a}}$$

Time period of circular motion

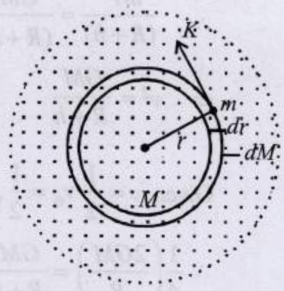
$$T=\frac{2\pi r}{v}=\frac{2\pi a/\sqrt{3}}{\sqrt{\frac{Gm}{a}}}=2\pi\sqrt{\frac{a^3}{3Gm}}$$

## Topic-5: Miscellaneous (Mixed Concepts) Problems

**1.** **(b)** Gravitational pull of the mass '$M$' present in the sphere of radius '$r$'. Provide the required centripetal force of particle of mass '$m$' to revolve in a circular path.

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow \frac{1}{2}mv^2 = \frac{GMm}{2r} \Rightarrow K = \frac{GMm}{2r}$$

$$\therefore \quad M = \frac{2Kr}{Gm}$$




Differentiating the above equation w.r.t '$r$' we get

$$\frac{dM}{dr} = \frac{2K}{Gm}$$

$$\text{or} \quad dM = \frac{2K}{Gm}dr$$

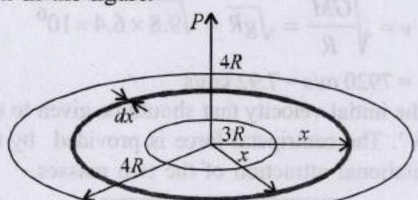
$$\therefore \quad 4\pi r^2 dr\, \rho = \frac{2K}{Gm}dr \Rightarrow \rho = \frac{K}{2\pi r^2 mG}$$

$$\therefore \quad \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G} \quad \text{or} \quad \frac{\rho(r)}{m} = \frac{K}{2\pi r^2 m^2 G}$$

**2.** **(a)** Mass per unit area of the shaded part.

$$\sigma = \frac{\text{mass}}{\text{area}} = \frac{M}{\pi((4R)^2 - (3R)^2)} = \frac{M}{7\pi R^2}$$

Let us consider a ring of radius $x$ and thickness $dx$ as shown in the figure.




Mass of the ring, $dM = \sigma 2\pi x dx = \dfrac{2\pi M x dx}{7\pi R^2}$

Potential at point $P$ due to shaded part

$$V_P = \int_{3R}^{4R} -\frac{G dM}{\sqrt{(4R)^2 + (x)^2}} = -\frac{GM 2\pi}{7\pi R^2}\int_{3R}^{4R}\frac{x dx}{\sqrt{16R^2 + x^2}}$$

Solving, we get

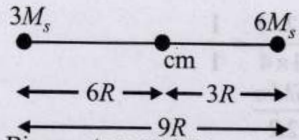
$$V_P = -\frac{GM 2\pi}{7\pi R^2}\left[\sqrt{16R^2 + x^2}\right]_{3R}^{4R} = -\frac{2GM}{7R}(4\sqrt{2} - 5)$$

Workdone in moving a unit mass from P to $\infty = V_\infty - V_P$

$$\text{or} \quad W_{P\infty} = 0 - \left(\frac{-2GM}{7R}(4\sqrt{2} - 5)\right) = \frac{2GM}{7R}(4\sqrt{2} - 5)$$

**3.** **(9)** The centre of mass lies at a distance $6R$ from lighter mass In circular orbit,

$$\text{Time period, } T = 2\pi\sqrt{\frac{R^3}{GM_S}}$$




Binary stars system

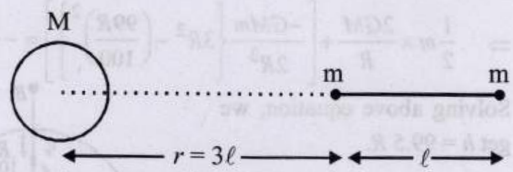
$$nT = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S + 6M_S)}}$$

$$\text{or,} \quad n \times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

$$\left[\because T = 2\pi\sqrt{\frac{R^3}{GM_S}}\right] \quad \therefore n = 9$$

**4.** **(7)** For point mass at distance $r = 4l$

$$\frac{GMm}{(4l)^2} + \frac{Gm^2}{l^2} = ma$$




For point mass at distance $r = 3l$

$$\frac{GMm}{(3l)^2} - \frac{Gm^2}{l^2} = ma$$

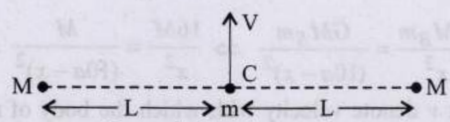
$$\therefore \quad \frac{GMm}{(4\ell)^2} + \frac{Gmm}{\ell^2} = \frac{GMm}{(3\ell)^2} - \frac{Gmm}{\ell^2}$$

$$\therefore \quad 2m = M\left[\frac{1}{9} - \frac{1}{16}\right] \Rightarrow m = \frac{7M}{288} \quad \therefore K = 7$$

**5.** **(b, d)** From conservation of mechanical energy,

$$\frac{-GMm}{L} - \frac{GMm}{L} + \frac{1}{2}mv^2 = 0 + 0$$

$$\text{or,} \quad \frac{1}{2}mv^2 = \frac{2GMm}{L} \quad \therefore \quad V = \sqrt{\frac{4GM}{L}} = 2\sqrt{\frac{GM}{L}}$$




Total energy of mass 'm' is conserved as there is no external force involved.